6D interpolation by incorporating angular weight constraints into 5D MWNI

Mark Ng, Xi-Shuo Wang and Dan Negut, Divestco Inc.*

Summary

Conventional 5D minimum weighted norm interpolation (MWNI) starts data fitting from the stable low frequencies, and recursively works its way up to high frequencies. This unconstrained fitting can lead to inaccurate results under challenging scenarios such as meager data support, up sampling of regularly missing data, or aliased dips. In this paper, we propose an additional dimension along multiangular directions to be added to the 5D MWNI in order to guide the a priori model in the frequency-wavenumber domain. Angular weights connect data information across all frequency-wavenumbers, which is key to de-aliasing of data, but is completely missing in the conventional 5D MWNI.

Introduction

The 5D interpolation method for 3D prestack seismic data by MWNI has been widely used since its introduction by Trad (2009, 2014), building on the work of Liu and Sacchi (2004). As mentioned in the above summary, there is no global data insight extracted from the frequency-wavenumber domain since an independent 4D MWNI is applied in space for every frequency slice, one slice at a time. Curry (2010) and Naghizadeh (2012) suggested imposing a Fourier-radial thresholding mask directly in data fitting, a method that is not MWNI based, and such a strong constraint could lead to results that are somewhat too smooth. Ng and Negut (2013, 2014) suggested integrating the Fourier angular stack concept into MWNI, and thereby raising the dimension of the MWNI by one. There, the angular weighted MWNI (AwMWNI) was applied to 2D prestack data. The natural extension here is to demonstrate how to apply the extra dimension of angular weight to 5D MWNI for 3D prestack data to get a 6D MWNI i.e. AwMWNI. A related method is that of Chiu (2013, 2014) which uses a model-constrained MWNI by dip scanning a few major dips in the time-space domain, a method that is essentially doing interpolation twice: once for the a priori model building, and once for the actual MWNI. But AwMWNI performs the angular weight 'scan' and the MWNI in the frequency-wavenumber domain both together in one step giving a speed advantage.

Theory and Method

In 3D prestack data, let $d(x,y,h,g,f)$ be the complete data in 5D space: inline x, crossline y, offset x indicated by h, offset y indicated by g, and f frequency; and its corresponding 4D spatial Fourier transform $D(k_x, k_y, k_h, k_g, f)$. So at any frequency f, the desired unknown complete data can be written as

$$d = F^H \overline{D} m \qquad (1),$$

where F^H is the 4D wavenumber inverse Fourier transform operator, $\overline{D}$ (i.e. $|D|$) the unknown 4D complete data amplitude spectrum, m the unknown 4D 'model' phase, H the conjugate transpose. However, what is given is the observed incomplete data which is masked by the known sampling matrix T,

$$Td = \left[TF^H \overline{D}\right] m \equiv [L] m \qquad (2).$$

By forming the adjoint operator, the model phase

$$m = [L]^H Td = \left[\overline{D} FT\right] Td \qquad (3)$$

can be found through data fitting by the conjugate gradient method. However, a reasonably close approximation of the complete amplitude spectrum $\overline{D}$ at each frequency slice is needed as the a priori $\overline{D}_0$ to initiate the inversion.

Conventional 5D MWNI simply uses the updated result of $\overline{D}$ from previous frequency as the a priori of current frequency without a foreknowledge of higher frequency, resulting in lacking a global insight. So under challenging data scenarios, $\overline{D}$ can be inaccurate and mislead the inversion thereafter. Here we propose a multi dimensional Fourier-radial angular weight gamma γ that conditions the known input unbiased incomplete data amplitude spectrum $|D_{in}|$ to get the a priori,

$$\overline{D}_0 = |D_{in}|\gamma^p = |FTd|\gamma^p \qquad (4),$$

where p is an arbitrary non negative power controlling gamma sharpness. If either $p = 0$ or $\gamma = 1$, there is no angular weight or global data insight, and equation (4) degenerates to conventional 5D MWNI. The angular weight gamma γ has a uniform amplitude in the Fourier-radial direction and is defined as

$$\gamma(k_x, k_y, k_h, k_g, f) = A(\theta_x, \theta_y, \theta_h, \theta_g) \qquad (5).$$

A is the angular sum of amplitude spectral values along the Fourier-radial direction of all usable apparent dip angle θ_i,

DOI http://dx.doi.org/10.1190/segam2015-5875338.1

$$A(\theta_x,\theta_y,\theta_h,\theta_g)=\int_r \left|D_{in}(\theta_x,\theta_y,\theta_h,\theta_g,r)\right| dr \qquad (6),$$

$$r=\sqrt{k_x^2+k_y^2+k_h^2+k_g^2+f^2},\ \theta_i=\tan^{-1}\frac{k_i}{f},\ i=x,y,h,g \quad (7).$$

The proposed method adds one extra angular dimension to the conventional 5D MWNI by connecting spectral information across all frequency-wavenumbers, which is crucial to the de-aliasing of dipping data but is totally missing in the conventional 5D MWNI. In doing so, 5D MWNI becomes 6D AwMWNI.

Example

A structural 3D prestack real data set is used to compare the performance of the proposed 6D AwMWNI with the conventional 5D MWNI in recovering missing data under a highly decimated situation. Upsampling structural data scenario is one of the most challenging recovery exercises for MWNI type or any other interpolation algorithm.

Figure 1 is an inline of a 3D stack of the complete prestack data being the 'hidden' control reference, and a subset of its amplitude spectrum $\overline{D(k_x,k_y,k_h,k_g,f)}$ i.e. $|D|$, at k_h=k_g=0 taken around the green circle area are shown below it. Since it is difficult to display a full 5D spectrum, just a 3D subset of it is shown. The data is 60-fold, trace spacing of 25 m. A window of 1 sec is shown. Conflicting diffraction curves with different amplitudes are present in both left and right sides of the figure.

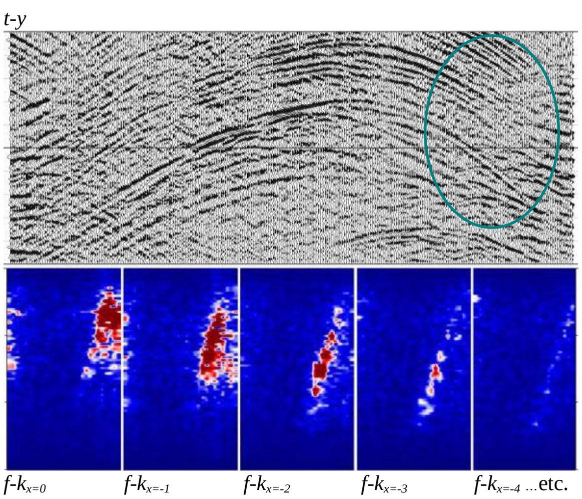

Figure 1. Full complete data stack of all gathers – the control reference with the corresponding *f-k* amplitude spectrum $|D|$ (at k_h=k_g=0) taken at the green circled area.

Figure 2 is the stack for input CDP gathers regularly 3:1 decimated in the crosslines. Consequently, only 33% of the original data are used, and are treated as input to the 5D and 6D interpolation tests. The steeply dipping diffraction curves become aliased inside two yellow highlighted circles. However, in the middle part of the stack, the gently curved diffractions are not aliased. The incomplete data amplitude spectrum $|D_{in}|$ at the right yellow circled area is shown below it.

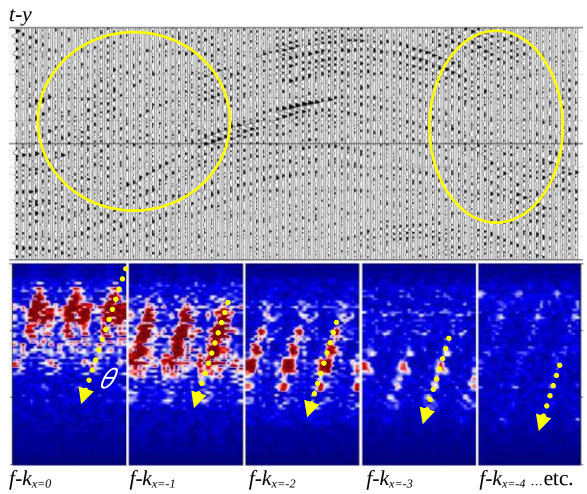

Figure 2. Stack of input 3:1 decimated CDP gathers. Data aliasing is observed inside the highlighted circles with the corresponding *f-k* amplitude spectrum $|D_{in}|$ (at k_h=k_g=0) taken at the right yellow circled area. The decimation factor of 3 causes the full spectrum to repeat itself three times with equal amplitude; and furthermore, the overlapping of energies due to steeply dipping data makes data recovery challenging.

The straight yellow dotted arrows in figure 2 spectrum will be explained latter after figure 3.

Figures 3 shows the data recovery results of conventional 5D interpolation by MWNI, and the corresponding recovered amplitude spectrum $|D|$. In order to preserve local structural details, 5D interpolation is done in small output data blocks of 500 ms windows, 10 by 10 CDPs, 27 by 27 offset traces with overlapping input data block size of 28 by 28 CDPs.

DOI http://dx.doi.org/10.1190/segam2015-5875338.1

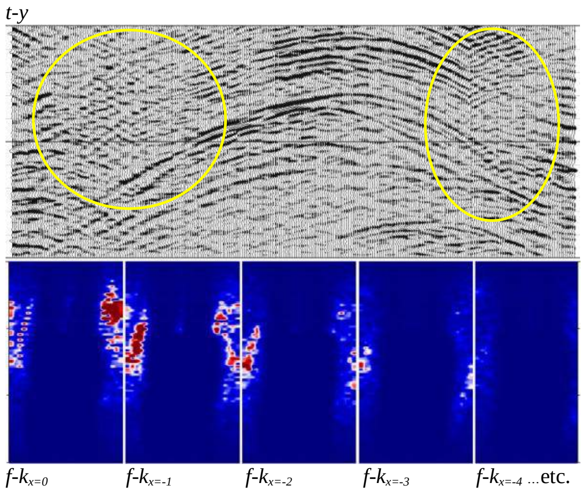

Figure 3. Stack of the recovery by conventional 5D MWNI. Poor quality recovery of aliased steeply dipping data is shown inside the yellow circles. The recovered amplitude spectrum $|D|$ resolves the dipping energies in a completely wrong direction deviating from the correct control reference spectrum provided in figure 1. This is due to the fact that frequency slice-by-slice MWNI inversion in the conventional 5D interpolation lacks the full ability of connecting dipping information across the frequency-wavenumber domain.

The core idea of the angular weight method is that dipping information is first derived from the input incomplete data spectrum through finding the angular sum *A*, and then second, it is imposed back to the input incomplete data spectrum to correct it by applying the angular weight gamma γ:

Referring back to the observed incomplete data spectrum in figure 2, the straight yellow dotted arrows indicate how one of many angular sums *A* is found by integrating along that Fourier-radial direction starting from origin using equation (6). It is essentially a dip scan scheme in the five dimensional frequency-wavenumber domain. When the angular search θ_i direction coincides with an actual dip angle, it will give a strong angular sum value for $A(\theta_x, \theta_y, \theta_h, \theta_g)$, being insensitive to the confusing spectral information, and not so much affected by it. The dotted arrows in fact can go beyond any Nyquist wavenumbers in k_x, k_y, k_h or k_g, and wrap around in order to connect the aliased energies of dipping events.

Figure 4 illustrates a subset of the fully measured 5D angular weight gamma $\gamma(k_x, k_y, k_h, k_g, f)$ at k_h=k_g=0 by populating the 4D angular sum *A* as described in equation (5). Then, the gamma is used to filter the input spectrum $|D_{in}|$ in order to give a 'best' estimation of the fully sampled data spectrum - the a priori $\overline{D_0}$ as described in equation (4) for 6D AwMWNI. When linear dips are not detected, gamma will approach unity ($\gamma = 1$), and equation (4) degenerates back to the 5D MWNI solution.

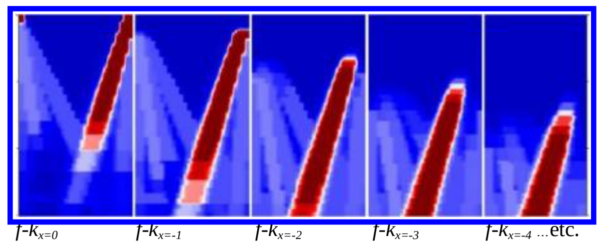

Figure 4. The angular weight gamma γ^p calculated from $|D_{in}|$ of the input incomplete data by equation (5). Power p of 2 is used to control the sharpness of gamma. Gamma reveals different dips with different strengths resembling the trends of the control reference complete data *f-k* spectrum in figure 1.

Figures 5 shows the data recovery results by the proposed 6D interpolation by AwMWNI, and the corresponding recovered amplitude spectrum $|D|$. Identical data blocking parameters are used as for the previous 5D method. In this example, the run time is about 30% more than that of the conventional 5D interpolation due to the angular weights calculation and application. The proposed 6D interpolation recovery is very acceptable when compared to the control reference.

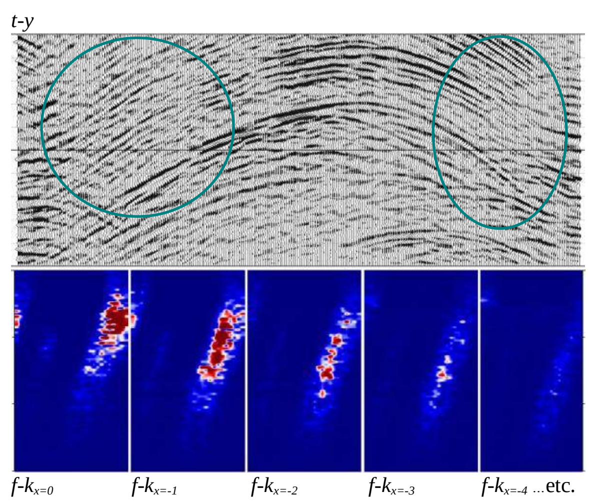

Figure 5. Stack of the recovery by the proposed 6D AwMWNI.

DOI http://dx.doi.org/10.1190/segam2015-5875338.1

It provides a reasonably good quality recovery overall and at the green highlighted circles in particular when compared to the control reference in figure 1, considering that the input data is 3:1 decimated, so deficient for such recovery. The recovered amplitude spectrum |*D*| has delineated the aliased energies yielding a result that is similar to that of the control reference.

Figure 6 reveals the stack differences between the full data control stack (figure 1) and the 5D MWNI recovery (figure 3). Data leakage is evident at the steep dip areas.

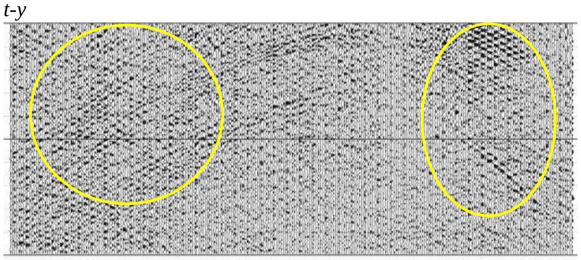

Figure 6. The difference stack of the conventional 5D interpolation. Large differences are seen at the steeply dipping diffractions that are aliased.

Figure 7 reveals the stack differences between the full data control stack (figure 1) and the 6D AwMWNI recovery (figure 5). Data leakage is less evident when compared to that of the 5D MWNI recovery.

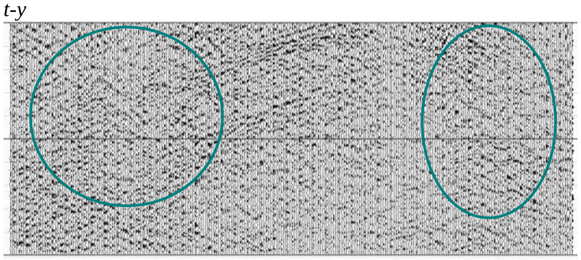

Figure 7. The difference stack of the proposed 6D interpolation. Much smaller differences are seen at the highlighted steeply dipping area when compared to that of the conventional 5D interpolation shown in figure 6.

Conclusions

In contrast to the conventional 5D interpolation by MWNI, the proposed 6D interpolation by AwMWNI has an additional angular weight function dimension that connects dipping data information across the all frequency-wavenumbers, which in effect can delineate aliased data in highly structural and deficient in data support situation. But when the data complexity is less challenging, and the data support is sufficient, the 5D and 6D interpolations will converge to similar results.

DOI http://dx.doi.org/10.1190/segam2015-5875338.1

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2015 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Chiu, S., 2013, 5D anti-aliasing interpolation: Application on an unconventional shale play: 83rd Annual International Meeting, SEG, Expanded Abstracts, 3608–3612, doi: 10.1190/segam2013-0308.1.

Chiu, S., 2014, Multidimensional interpolation using a model-constrained minimum weighted norm interpolation: Geophysics, **79**, no. 5, V191–V199. http://dx.doi.org/10.1190/geo2014-0086.1.

Curry, W., 2010, Interpolation with Fourier-radial adaptive thresholding: Geophysics, **75**, no. 6, WB95–WB102. http://dx.doi.org/10.1190/1.3500977.

Liu, B., and M. D. Sacchi, 2004, Minimum weighted norm interpolation of seismic record: Geophysics, **69**, 1560–1568. http://dx.doi.org/10.1190/1.1836829.

Naghizadeh, M., 2012, Seismic data interpolation and denoising in the frequency-wavenumber domain: Geophysics, **77**, no. 2, V71–V80. http://dx.doi.org/10.1190/geo2011-0172.1.

Ng, M., and D. Negut, 2013, Data interpolation by angular weighted MWNI: Annual GeoConvention Meeting.

Ng, M., and D. Negut, 2014, Minimum weighted norm interpolation with an angular-weighted deconvolved prior: 84th Annual International Meeting, SEG, Expanded Abstracts, 3658–3662, doi: 10.1190/segam2014-0573.1.

Trad, D., 2009, Five-dimension interpolation: Recovering from acquisition constraints: Geophysics, **74**, no. 6, V123–V132. http://dx.doi.org/10.1190/1.3245216.

Trad, D., 2014, Five-dimension interpolation: New directions and challenges: CSEG Recorder, **39**, no. 3, 22–29.